

# OPTIMIZATION OF NOISE PERFORMANCE FOR VARIOUS TOPOLOGIES OF MICROWAVE ACTIVE RECURSIVE FILTERS

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## Abstract :

This article deals with the improvement of the noise figure for three topologies of microwave active recursive filters. We show how, using a noise wave formalism [1], the noise factor of these filters can be effectively minimized by using appropriate unbalanced power dividers/combiners and an amplifier. A comparison between the different topologies is given. We start from measured results and validate our approach with simulated examples.

## I - Introduction

With the rapid expansion of new applications, such as mobile communications, microwave engineers have found great advantage in using active filters. Nevertheless, the use of active elements in microwave systems has introduced new design parameters and problems such as electrical stability, power handling behavior and noise figure. Among all the existing structures, recursive and transversal filters have recently appeared as a promising solution to filtering problems. A first-order filter has been implemented [2] on a 100 $\mu$ m-thick GaAs substrate using a MMIC design process (figure 1). Since noise performances were not considered during the different design steps, we obviously obtained a poor noise factor of 11.7dB (figure 2).

The objective here is to find which of the three possible topologies achieves the best noise performance by analytically calculate the noise factors using a noise wave formalism. For each topology, we also determine the values of the optimal couplings and of the gain needed to obtain the minimum noise factor.

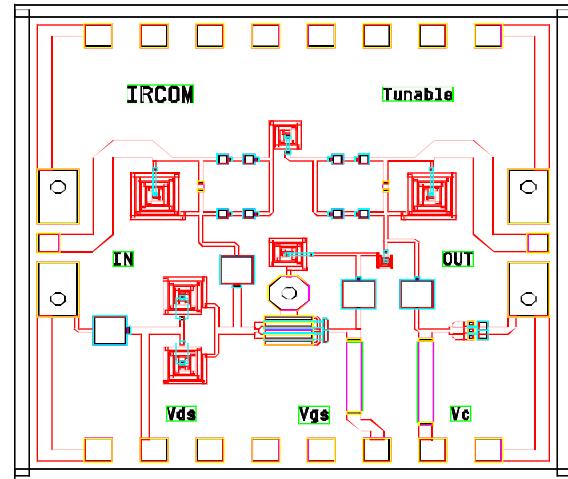


Figure 1 : Layout of the first-order active recursive filter (GaAs substrate 2x2 mm)

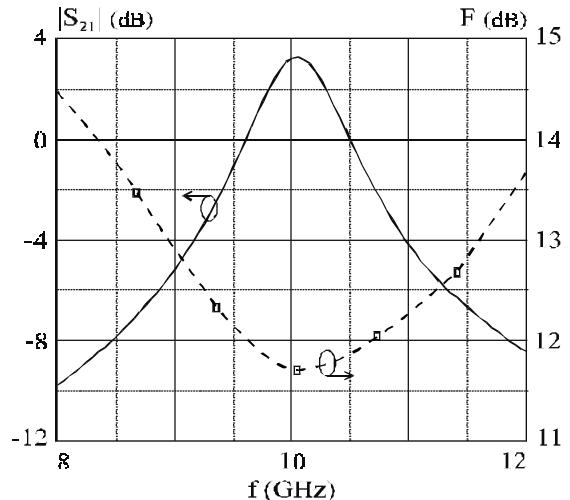


Figure 2 : Measured  $S_{21}$  and noise figure of a MMIC first-order recursive filter

## II-Minimum noise factor for three topologies of microwaves active recursive filters

Feasibility of first-order recursive filters is derived from digital low frequency concepts. Expression (1),

where  $x(t)$  [ $y(t)$ ] is the input [output] of the system, shows the time-domain equation of a first-order structure :

$$y(t) = a_0 x(t) - b_1 y(t - \tau) \quad (1)$$

The corresponding transfer function in the z-notation is given by :

$$H(Z) = \frac{a_0}{(1 + b_1 Z^{-1})} \quad \text{where } Z = e^{-2j\pi f\tau}$$

$\tau = 1/f_0$  is the delay-time of the filter and  $f_0$  is the central frequency.

To calculate the noise factor, we use the noise wave formalism of [1]. This method consists in modeling the noise of a two-port device with internal noise wave generators  $c_1$  and  $c_2$ . Their contribution to the scattering waves  $b_1$  and  $b_2$  can be expressed in the following manner :

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}_Q \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

where  $S$  is the scattering matrix of the two-port  $Q$ .

To calculate the minimum noise factors of the three topologies, we first define the  $R$  and  $V$  parameters to set the filtering performances of the filter which must not be modified by the improvement process of the noise factor.

$$R = \frac{|S_{21}|_{\max}}{|S_{21}|_{\min}} = \frac{|S_{21}(f_0)|}{|S_{21}(f_0/2)|} \quad \text{and} \quad V = \frac{R-1}{R+1}$$

where  $f_0$  is the central frequency of the filter.

In order to get the same filtering performances, the three topologies have to be characterized by the same  $V$  parameter. Two filters can then be considered as equivalent even if they do not achieve the same gain at  $f_0$ . Referring to the  $V$  parameter, the transfer function of the three topologies can then be expressed as follows :

$$H_i(Z) = \frac{K_i}{1 - VZ^{-1}}$$

### II.1 - Topology 1 : (figure 3)

Initially, we suppose  $G = G_0 e^{-2j\pi f\tau}$  with  $G_0$  a positive real gain value and  $\tau$  the delay-time of the filter. The transfer function is given by :

$$H_1(f) = \frac{\alpha_1 \alpha_2}{1 - G \beta_1 \beta_2}.$$

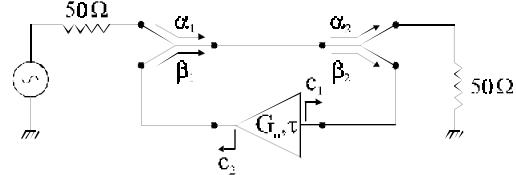


Figure 3 : *Topology 1*  
Amplifier placed within the feedback branch

At  $f_0$ , the central frequency of the filter, the noise factor is given by (2), with  $F_A$  is the noise factor of the amplifier derived from the same noise wave formalism and  $b_g$  is the noise wave due to the source resistance at the input port of the filter.

$$F_1 = 1 + \left( \frac{\beta_1}{\alpha_1} \right)^2 (F_A - 1) G_0^2 + \left( \frac{\beta_2 - \beta_1 G_0}{\alpha_1 \alpha_2} \right)^2 \quad (2)$$

In this case,  $V = \beta_1 \beta_2 G_0$ ,  $\alpha_1^2 + \beta_1^2 = 1$ ,  $\alpha_2^2 + \beta_2^2 = 1$ . We can find an expression of  $F_1$  as a function of  $V$ ,  $\beta_1$ ,  $G_0$  and  $F_A$ . An analytical study enables us to determine the expression of  $\beta_{1\text{opt}}$ , for which we obtain the minimal noise factor. We can then deduce  $\beta_2$ ,  $\alpha_1$ , and  $\alpha_2$ . It can be shown that for  $\beta_1 = \beta_{1\text{opt}}$ ,  $F_1$  decreases when  $G_0$  increases. So the limit value of  $F_1$ , when  $F_A$  is kept constant, is given by :

$$\lim_{G_0 \rightarrow \infty} F_1 = 1 + V^2 (F_A + 1) - 2V + 2V\sqrt{F_A (1 - V)}$$

$$\text{and} \quad \lim_{G_0 \rightarrow \infty} F_1 < F_A$$

This is particularly interesting in the case where we can choose between two amplifiers of same noise factor but with different gain values. Moreover we can obtain, at the resonance frequency  $f_0$ , a noise factor of the filter lower than the one of the amplifier used.

### II.2 - Topology 2 : (figure 4)

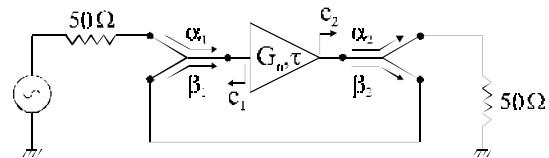


Figure 4 : *Topology 2*  
Amplifier placed within the forward branch

The transfer function  $H_2(f)$  and the corresponding noise factor  $F_2$  are given by :

$$H_2(f) = \frac{G \alpha_1 \alpha_2}{1 - G \beta_1 \beta_2}$$

$$F_2 = 1 + \frac{(F_A - 1)}{\alpha_1^2} + \left( \frac{\beta_2 - \beta_1 G_o}{\alpha_1 \alpha_2 G_o} \right)^2$$

With the same method than for topology 1, we find that the expression of  $\beta_{1\text{opt}}$  is the same as for topology 1 but with  $F_2 > F_1$  at  $f_0$ . We notice :

$$\lim_{G_0 \rightarrow \infty} F_2 = F_A$$

and that the noise factor of the filter cannot be less than  $F_A$ .

### II.3 - Topology 3 : (figure 5)

The corresponding transfer function  $H_3(f)$  and the noise factor  $F_3$  are given by :

$$H_3(f) = \frac{G\alpha_1\alpha_2}{1 - \beta_1\beta_2 e^{-2j\pi f\tau}} = GH(f)$$

$$F_3 = F_A + \frac{1 - |H(f)|^2}{G_0 |H(f)|^2}$$

where  $H(f)$  is the transfert function of a passive filter. In this case  $V = \beta_1 \beta_2$ . With the same method we obtain :

$$\beta_{1\text{opt}} = \sqrt{V} \Rightarrow \beta_{2\text{opt}} = \frac{V}{\beta_{1\text{opt}}} = \sqrt{V} = \beta_{1\text{opt}}$$

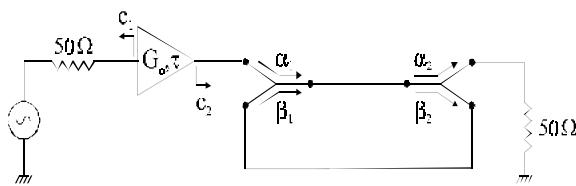


Figure 5: Topology 3  
Amplifier cascaded with a passive filter

$F_3$  is minimum when the couplers are identical. Moreover for this topology :

$$\lim_{G_0 \rightarrow \infty} F_3 = F_A$$

leading to the same remarks than for topology 2.

### **III - Comparison of the three topologies**

To compare the topologies, we consider that the same amplifier is used with the same noise factor and gain values and we also consider that the three topologies are optimum in terms of noise ( $\beta_1 = \beta_{1\text{opt}}$ ) and achieve the same filtering performance  $V$ . It can be analytically derived, thanks to the software Maple V [3], that :

$$\begin{cases} F_3 < F_1 < F_2 & \text{if } 1 < G_o < G_{\lim} \\ F_1 < F_3 < F_2 & \text{if } G_o > G_{\lim} \end{cases}$$

$$\text{with : } G_{\lim} = \frac{V}{(1-V)} \sqrt{\frac{F_A(V+1) + (1-V) + 2\sqrt{F_A}}{F_A(V+1)^2 - (V-1)^2}}$$

for which  $F_3 = F_1 = F_2$  at  $f_0$ .

We can then say that topology 2 is the worst topology for noise and that the choice between topology 1 and 3 depends on  $V$  and the gain  $G_o$  of the amplifier. Besides, an important advantage of topology 1 is that this topology is the most interesting regarding to power handling behavior because the amplifier is placed within the feedback branch and see at its input less power. So topology 1 can be considered as the best solution for our problem.

## IV - Validation examples

Using a microwave CAD software, we present three examples to validate our calculations.

**IV.1 - Example 1 :  $G_0 = 1.316$ ,  $F_A = 9.3$  dB,  $R = 5$  ( $V = 2/3$ ),  $G_{lim} = 1.2$ ,  $G_0 > G_{lim}$**

The values of  $G_o$ ,  $F_A$  and  $R$ , correspond to the ones used in the filters already realized using MMIC technology (figures 1 and 2). We notice that the optimum noise factor decreases down to 8.47 dB less than  $F_A$  (table 1).

	$\beta_{1\text{opt}}$ (dB)	$\beta_2$ (dB)	$F(f_0)$ (dB)	simulated results
topology 1	-5.12	-0.76	8.46	figure 6
topology 2	-5.12	-0.76	10.75	
topology 3	-1.76	-1.76	9.28	

Table 1 :  $G_o > G_{lim} \Rightarrow F_1 < F_3 < F_2$  at  $f_o$  \$

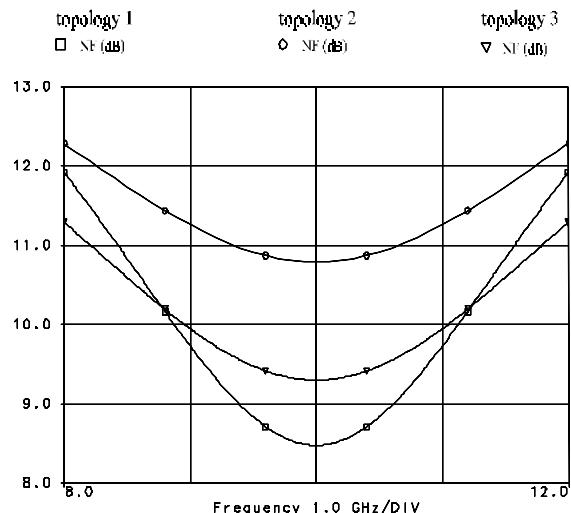


Figure 6: Simulated noise figures for example 1 :  $G_o > G_{lim} \Rightarrow F_1 < F_3 < F_2$  at  $f_0$

#### IV.2 - Example 2 : $G_o = 1.316$ , $F_A = 9.3$ dB, $R = 10$ (V = 0.82), $G_{lim} = 1.8$ , $G_o < G_{lim}$

We now try to built more selective filters with the same amplifier. We set  $R=10$  (V=0.82), and in this case  $G_o < G_{lim}$  leading to  $F_3 < F_1 < F_2$  (table 2) as expected. With examples 1 and 2, we also notice that the noise factors increase when selectivity increases.

	$\beta_{1opt}$ (dB)	$\beta_2$ (dB)	$F(f_0)$ (dB)	simulated results
topology 1	-3.88	-0.25	10.2	
topology 2	-3.88	-0.25	11.44	
topology 3	-0.87	-0.87	9.35	figure 7

Table 2 :  $G_o < G_{lim} \Rightarrow F_3 < F_1 < F_2$  at  $f_0$

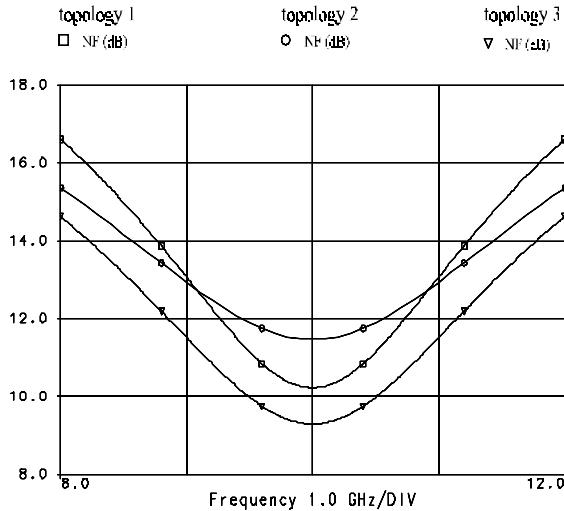


Figure 7: Simulated noise figures for example 2 :  $G_o < G_{lim} \Rightarrow F_3 < F_1 < F_2$  at  $f_0$

#### IV.3 - Example 3 : $G_o = 2$ , $F_A = 9.3$ dB, $R = 10$ (V = 0.82), $G_{lim} = 1.8$ , $G_o > G_{lim}$

In this case, we keep  $R=10$  but we increase the gain ( $G_o = 2$ ), which leads to  $G_o > G_{lim}$ . We obtain  $F_1=8.91$  dB,  $F_2=10.07$  dB and  $F_3=9.28$  dB (figure 8). This shows that if the gain is increased (for the same  $F_A$ ) the noise factor of the filter decreases as expected.

### Conclusion

In this article, we have analytically analyzed three different topologies of microwave active recursive filters using the same active elements. We have derived, using a noise wave formalism, the analytical expressions of the noise factor of each topology. We

have also shown the existence of optimal coupling values, which achieve minimum noise factors.

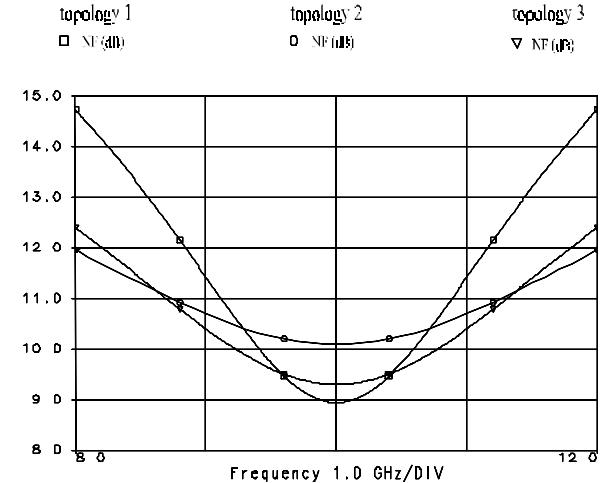


Figure 8: Simulated noise figures for example 3 :  $G_o > G_{lim} \Rightarrow F_1 < F_3 < F_2$  at  $f_0$

The most important result is that with the first of the three topologies, when the amplifier is placed in the feedback branch, the noise factor of the filter could be less than the one of the amplifier used. This result shows that recursive structures are promising solution to low noise filtering problem even better than the classical cascade approach of an amplifier with a passive filter structure. We have also studied the influence of the value of the gain upon the noise factor of the three topologies and validated our analysis and results with different simulated examples.

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